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PROJECT BOOKLET

MAT2038 Applied Mathematics 20
Module 2

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Module
2

Applied Mathematics 20

NON-LINEAR FUNCTIONS

PROJECT BOOKLET

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FOR TEACHER'S USE ONLY

Summary

Total Possible Marks	Your Mark
35	

Teacher's Comments

Applied Mathematics 20
Module 2: Non-Linear Functions
Project Booklet
Learning Technologies Branch
ISBN 0-7741-1907-1

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PROJECT BOOKLET
APPLIED MATHEMATICS 20: MODULE 2

Your mark for this module will be determined by how well you do on the module project in this Project Booklet and the module assignment in the Assignment Booklet.

The value of each part of the module project is stated in the left margin of this booklet. The total value of the module project is 35 marks.

Read all parts of this project carefully and record your answers in the appropriate place. Work slowly and carefully. If you are having difficulities, go back and review the appropriate activity in the Student Module Booklet.

Be sure to complete all parts of the project and proofread your responses before submitting this project to your teacher. If you require more room for any response, use your own paper and attach it securely to this booklet.

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Module Project: The World's Population

Your project for Module 2: Non-Linear Functions is The World's Population. This project involves using models for the world's population growth; comparing the validity of these models; and writing a short report indicating historical growth, predictions about future growth, and the impact of continued growth on Earth's resources.

1. Turn to pages 82 and 83 of the textbook and answer exercise 2.a. to 2.e., 2.g., and 2.h. of "Modelling the Growth of the World's Population." Note that the label "1996" on the graph on page 85 should read "1990."

① **Textbook exercise 2.a.:** ______________________________

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① **Textbook exercise 2.b.:** ______________________________

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① **Textbook exercise 2.c.:** ______________________________

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① **Textbook exercise 2.d.:** ______________________________

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(½) **Textbook exercise 2.e.:** ______________________________

____

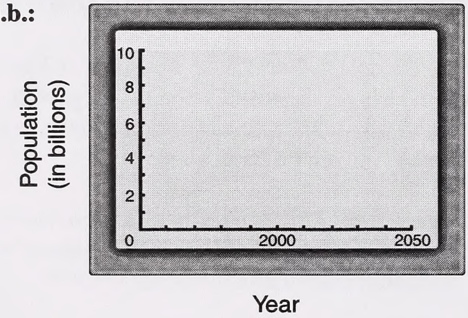
($\frac{1}{2}$) **Textbook exercise 2.g.:**

① **Textbook exercise 2.h.:**

2. Turn to page 83 of the textbook and answer exercise 3 of “Modelling the Growth of the World’s Population.” For exercise 3.d., use the Intersect feature.

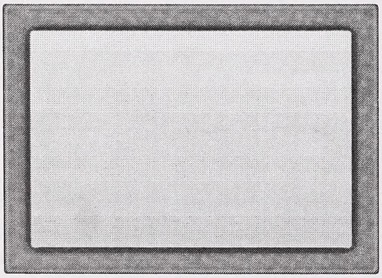
① **Textbook exercise 3.a.:**

① **Textbook exercise 3.b.:**



② **Textbook exercise 3.c.:**

② **Textbook exercise 3.d.:** (Sketch what your calculator shows on the blank display given. You may wish to use the features in the CALCULATE menu instead of the Trace feature.)



④ **3.** Turn to page 84 of the textbook and answer exercise 4 of “Modelling the Growth of the World’s Population.”

① **4.** Refer to the table in exercise 4 on page 84 of the textbook and to the graph on page 85. Both the table and the bar graph show a drop in the growth rate.

When using an exponential model for population growth, what assumption about the growth rate are you using?

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If your graphing calculator can run the WORLDPOP program, do exercise 5. If your graphing calculator cannot run this program, do exercise 6.

5. Turn to page 104 of the textbook and read the first two paragraphs of "A Logistic Model for the Growth of the World's Population."

Use the WORLDPOP program with the 1980 figures from the table on page 84 of the textbook.

Turn to pages 104 and 105 of the textbook and work through the instruction preceding exercise 1; then do the following questions.

④ **a.** Use the WORLDPOP program to create graphs for $M = 10$ and $M = 20$. Sketch the resulting graph for each value of M.

$M = 10$ **$M = 20$**

④ **b.** Use the table of values, the features from the CALCULATE menu, or the Trace feature to predict the population in 2000, 2010, 2020, and 2100 for $M = 10$ and $M = 20$. Create a table to display your results.

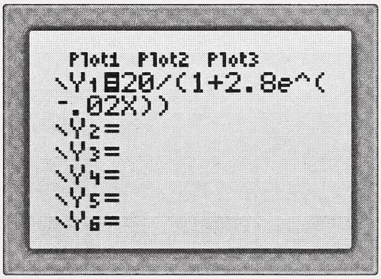
④ **c.** How do the predicted populations for $M = 20$ in exercise 5.b. compare with that of the student's predictions using the 1960 data on page 109 of the the textbook?

6. A good model should take into account the maximum population capacity of Earth. One such model is the following function:

$$y=\frac{20}{1+2.8e^{-0.02x}}$$

The independent variable, *x*, is the time (in years) after 1990. The dependent variable, *y*, corresponds to the population (in billions). The variable *e* represents a special irrational number; it is approximately equal to 2.1723.

Enter this function into your graphing calculator's equation editor. **Note:** Enter *e* by pressing 2nd [e^x].



```
Plot1 Plot2 Plot3
\Y1=20/(1+2.8e^(
-.02X))
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

This model corresponds to the 1990 figures from the table on page 84 of the textbook, where the initial population is 5.28 billion and annual growth rate is 1.55%. The model also incorporates a maximum population of 20 billion.

③ **a.** Graph the world population for the period 1990 to 2190. Set $\text{Xmin} = 0$ and $\text{Xmax} = 200$, and choose appropriate values for Ymin and Ymax. When drawing the graph, include the labels and scales for the horizontal and vertical axes.

③ **b.** Use the calculator's features to determine the population in 2000, 2010, and 2100. Organize your results in a table.

⑥ **c.** Turn to page 84 of the textbook and refer to the table in exercise 4.

The following chart shows three functions that closely fit the population growth based on the textbook's figures, but reflect different maximum populations. Use the features on your graphing calculator to complete the chart with projected populations for 2000, 2010, 2020, and 2100. Round your answers to 2 decimal places.

The chart indicates models for the 1950 figures, where the initial population is 2.56 and the growth rate is 1.47%.

Year	Predicted Population Based on 1950 Figures (in billions)		
	M = 8 billion	*M* = 18 billion	*M* = 28 billion
	$y = \dfrac{8.002}{1 + 2.1318\, e^{-0.021\,68\,x}}$	$y = \dfrac{18.01}{1 + 6.0629\, e^{-0.017\,132\,x}}$	$y = \dfrac{28.02}{1 + 10.0004\, e^{-0.016\,158\,x}}$
2000			
2010			
2020			
2100			

⑥ **7.** Write a short report on the world's population growth. In your report, describe how the population has grown in the past; indicate the likely effect of the continued rapid population growth on Earth's resources; describe factors that can limit the rate of population growth; describe mathematical models that you have used to predict future populations, and indicate how accurately these models predict the future world population; and make a prediction of your own about future population growth.

(There is more room given on the next page.)

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ASSIGNMENT BOOKLET

MAT2038 Applied Mathematics 20
Module 2

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Module

2

Applied Mathematics 20

NON-LINEAR FUNCTIONS

ASSIGNMENT BOOKLET

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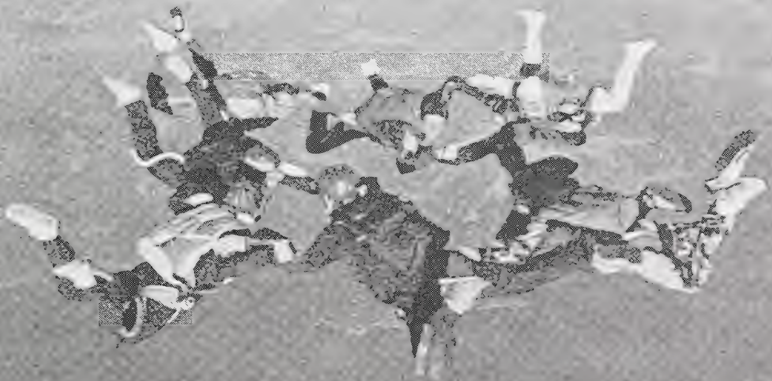

[redacted] [redacted]

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Summary

Total Possible Marks	Your Mark
65	

Teacher's Comments

Applied Mathematics 20
Module 2: Non-Linear Functions
Assignment Booklet
Learning Technologies Branch
ISBN 0-7741-1906-3

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APPLIED MATHEMATICS 20: MODULE 2

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The value of each part of the module assignment is stated in the left margin of this booklet. The total value of the module assignment is 65 marks.

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Be sure to complete all parts of the assignment and proofread your responses before you submit this assignment to your teacher for grading.

(65)

Module Assignment

②

1. Give an example of an exponential function. Write your example as an equation in terms of x and y. Explain why your example is an exponential function.

②

2. Explain the difference between exponential growth and decay.

3. A paramecium is a small, unicellular organism that can be found in local lakes and sloughs. Paramecia reproduce by splitting in two. In a laboratory flask, a colony of paramecia had an initial population of 500. Each day, the population of the paramecia was counted. The results are listed in the table on the right.

Time (in days)	Population
0	500
1	600
2	720
3	864
4	1037
5	1244
6	1493
7	1792
8	2150

③ **a.** Use your graphing calculator to make a scatterplot of the data in the table. Sketch the scatterplot you obtain on the screen given. Label the axes and show a scale for each axis on your sketch.

② **b.** Determine an exponential equation to represent the population as a function of time. Do not use your graphing calculator here.

② **c.** Suppose the flask and food supply is large enough to support the trend of the population growth. Estimate the population of the colony when the time is 10 days.

4. The balance of a savings account increased according to the table on the right.

Time (years)	Balance ($)
0	150.00
1	158.25
2	166.95
3	176.14
4	185.82
5	196.04
6	206.83

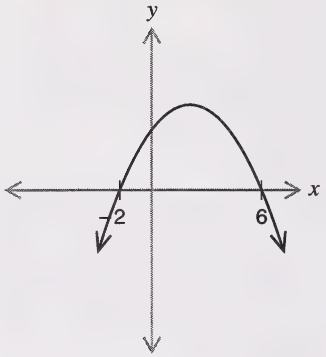
③ **a.** Graph the data in the table using your calculator. Sketch the graph you obtain on the screen given. Label the axes and show a scale for each axis on your sketch.

② **b.** Determine an exponential equation to represent the balance of the savings account as a function of time.

② **c.** Indicate the number of years the account would have to grow in order for the initial investment to double. State which calculator feature you used.

② **d.** Suppose the savings account was started with an initial investment of $600. How long would it take for this investment to increase to $1200? Explain how you obtain your answer without doing any calculations.

5. Use the graph of the quadratic function given to answer the following questions.



② **a.** What is the equation of the axis of symmetry of the graph? Explain your answer.

____

____

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① **b.** The maximum value of the equation is 5. Give the ordered pair representing the vertex of the graph.

____

____

① **c.** Indicate the range of the function.

____

② **6.** The axis of symmetry of the graph on the right is the vertical line $x = 5$. One of the x-intercepts is $(11, 0)$. What is the other x-intercept of the function? Explain your answer.

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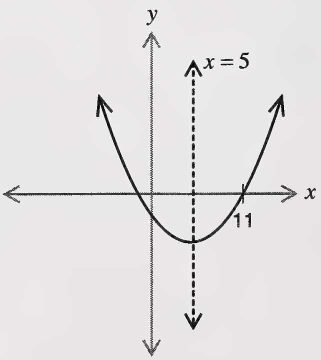


① **7.** Which of the following is a quadratic equation?

A. $x^2 + y^2 = 25$

B. $y = 3x^2 - 10x + 9$

C. $y = \dfrac{x}{x^2 + 1}$

D. $y = \dfrac{x - 1}{x^2 + 1}$

8. Graph the equation $y = 2x^2 - 6x - 36$ on your graphing calculator. Choose window settings so that the vertex and the *x*- and *y*-intercepts appear on the screen.

② **a.** Sketch the graph as it appears on your calculator screen. Include labelled axes with scales in your sketch.

① **b.** Determine the *x*-intercepts.

____

① **c.** What is the equation of the axis of symmetry?

____

① **d.** Determine the vertex.

____

② **e.** What is the domain and range of the function?

____

____

9. A long jumper reached a maximum height of 0.7 m when jumping a distance of 6.4 m measured from the start of his jump. Assume the jumper follows a parabolic trajectory.

① **a.** Give the coordinates of three points that you can infer from the given information to be on the trajectory.

② **b.** Use the Quadratic Regression feature on your graphing calculator to determine a function that best fits the parabolic trajectory of the jumper. (The function should give the height as a function of the distance travelled.) Round the coefficients to 3 decimal places.

② **c.** Graph the trajectory on your graphing calculator. Label the axes and indicate scale.

10. A promoter found that when organizing concerts at an 1800-seat hall, the revenue from ticket sales varied according to the price of the ticket. Based on several ticket prices, the promoter compared the revenue of several similar events. Here are the results.

Ticket Price ($)	Revenue ($)
30	45 000
40	50 000
45	50 625
48	50 353
55	48 125

② **a.** Using the Quadratic Regression feature, determine the quadratic function that best fits the data. Round the coefficients to 1 decimal place.

② **b.** Between what two ticket prices must the tickets be sold in order for the revenue to be at least $50 000 for an event. Round your answer to the nearest cent.

② **c.** Determine the ticket price that will maximize revenue. Round your answer to the nearest dollar.

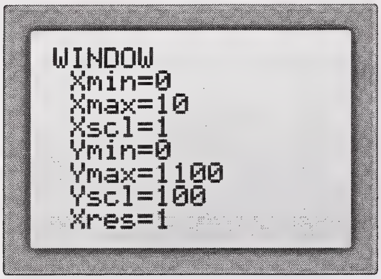
11. Starting from when a grasshopper is released, it jumps at the end of every minute. It is seen jumping towards a blade of grass. At the end of each minute, it jumps close to one-quarter of the distance remaining to the blade of grass. The data is summed up in the following table.

Time (min)	Distance Remaining (cm)
0	4
1	768
2	576
3	432
4	324
5	243

① **a.** Which regression curve best fits the data?

② **b.** Using the appropriate regression feature on your calculator or using algebraic reasoning, determine the function that models the data most closely.

② **c.** Graph the function you have determined to model the data. Select the windows setting as indicated. Sketch and label the graph that appears on your screen.



```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=1
 Ymin=0
 Ymax=1100
 Yscl=100
 Xres=1
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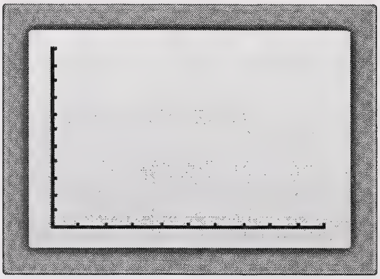

① **d.** Indicate the time it takes the grasshopper to be 76.89 mm from the blade of grass.

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____

② **e.** How long will it take the grasshopper to land on the blade of grass? Explain your answer.

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④ **12.** Solve the non-linear equation $x^2 = 3x + 4$ graphically using your calculator. Sketch and write the equation(s) of the graph(s) you used to find the solution.

④ **13.** The equation $5000 = 250(1.055)^n$, where n is the number of years, applies to an investment enjoying an interest rate of 5.5%, compounded annually. The solution of the equation indicates the time for an investment of \$250 to grow to \$5000. Solve the equation graphically using your calculator. Sketch and write the equation(s) of the label which graph(s) you used to arrive at the solution.

④ **14.** A paper strip is ripped in half and the halves are stacked on each other; the stack is then ripped in half and the resulting halves stacked on each other; and so on. The equation $2.5 = 0.005(2)^x$ can be solved to determine how many rips must be made (starting with the strip and then continuing with the stacks) to make a stack 2.5 cm high, given a paper strip is 0.005 cm thick.

Determine how many rips must be made by solving the equation $2.5 = 0.005(2)^x$ graphically using your calculator. Sketch and write the equation(s) of the graph(s) you used to arrive at the solution.

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